

Final Report

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## COMPUTER MODEL FOR LARGE ARRAYS OF WIND TURBINES

By

C. G. Justus

A. S. Mikhail

Prepared for

The United States Department of Energy  
Division of Distributed Solar Technology

Under

Contract No. EY-76-S-06-2439

May 1979

**GEORGIA INSTITUTE OF TECHNOLOGY**

**SCHOOL OF GEOPHYSICAL SCIENCES**

**ATLANTA, GEORGIA 30332**

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## SUMMARY

A synthesis of the earlier studies of wind and power distributions for large arrays of wind turbines is formulated in a computer model program. The program can be run with interactive (computer terminal) or batch input (card files, etc.). Based on a generic wind turbine power model, input characteristics at a single representative site, and data specifying the size of the array to be simulated, the program computes wind speed and wind power output statistics for the model array. Wind data for the input single site can be at 10 m level or at hub height (if the former, a height projection to hub height is performed by the program). Time series wind speeds at the single input site can also be used by the program to evaluate equivalent time-series wind speeds and output powers for the array. A program listing and input/output documentation are included.

## TABLE OF CONTENTS

<u>Section</u>	<u>Page</u>
Summary . . . . .	iii
List of Figures . . . . .	v
List of Tables . . . . .	vi
1. INTRODUCTION . . . . .	1
2. THE ARRAY MODEL PROGRAM . . . . .	4
3. SAMPLE PROGRAM OUTPUT . . . . .	17
References . . . . .	23
APPENDIX A - ARRAY MODEL PROGRAM LISTING . . . . .	24
APPENDIX B - THE TIME SERIES ARRAY MODEL . . . . .	31
APPENDIX C - COMPARISON WITH CLIFF ET AL. ARRAY MODEL . . . . .	36

## LIST OF FIGURES

<u>Figure</u>	<u>Title</u>	<u>Page</u>
1	An Example Power Output Curve Versus Wind Speed . . . . .	5
2	The Power Coefficient Curve Corresponding to Figure 1. . . . .	6
3	Schematic Map of an Array of n Sites (Farms) of Wind Turbines. . . . .	10
4	Sample Wind Speed Distribution Output, (from data in Table 3). . . . .	22
B-1	Probability $p( P(t + \Delta t) - P(t)  \leq \Delta P)$ for Time Change $\Delta t = 3$ Hours, Northeast Array. . . . .	33
B-2	Probability $p( P(t + \Delta t) - P(t)  \leq \Delta P)$ for Time Change $\Delta t = 12$ Hours, Northeast Array. . . . .	34
B-3	Probability $p( P(t + \Delta t) - P(t)  \leq \Delta P)$ for Time Change $\Delta t = 24$ Hours, Northeast Array. . . . .	35
C-1	Theoretical and Observed Ratio $\sigma_a/\bar{V}^{0.54}$ Versus Correlation $\rho$ . . . . .	39

## LIST OF TABLES

<u>Table</u>	<u>Title</u>	<u>Page</u>
1	Typical Surface Roughnesses . . . . .	8
2	Sample Array Program Output, Interactive Input . . . . .	18
3	Sample Array Program Output, Batch Input . . . . .	21

## 1. INTRODUCTION

The benefits of wind diversity to the operation of large arrays of wind turbines have been examined in several regions of the United States (Justus, 1976; Justus and Hargraves, 1977). These results and tests have been used to develop a computer model whereby the wind and wind power statistics can be estimated for arrays of arbitrary spatial size (Justus and Mikhail, 1978a).

Some of the procedures of this program were developed in separate studies. These include: the height projection method which depends on mean speed and surface roughness (Justus and Mikhail, 1976; Justus, 1978), the method for generic power output estimation (Justus and Mikhail, 1978b), and the method for estimating typical variation of Weibull distribution parameters with mean wind speed (Justus et al., 1976).

All of the development and testing of the array model has been documented in the references listed above. In this report the resulting model is described as a step-by-step procedure and is presented in the form of an interactive computer program for use in estimating wind speed and power statistics for arrays of arbitrary size. A listing of this program appears in Appendix A. One aspect of the model has not previously been documented: the time series simulation of array output from single site wind input. This documentation and testing is given in Appendix B.

A similar, but slightly different, modeling approach to wind speeds for arrays of sites has been reported by Cliff et al. (1978). Appendix C outlines the similarities and differences between these two models, and develops the necessary background for selection of one of the necessary model parameters not documented in the Cliff et al. report.





The following section describes the mathematical procedures employed in the Array Model computer program. A set of sample input and output is described in Section 3.

## 2. THE ARRAY MODEL PROGRAM

The following gives a mathematical description of the steps used in the array model program to compute distributions and statistics of array wind speed and array power from input values of wind speed for a single "representative" site. The program listing is given in Appendix A. A sample set of input and output are given in Section 3.

### Power Model Input to the Program

Certain characteristics of the wind turbine design whose array power is to be evaluated must be provided as input to the program. At a minimum, these parameters include the rated speed ( $V_r$ ) and the rated power ( $P_r$ ). If power coefficients A and B, described later, are not known, then the program also requires input values for the wind turbine cut-in speed ( $V_{in}$ ), cutout speed ( $V_{out}$ ), a value for the power coefficient ratio ( $C_{pr}/C_{pm}$ ), and an indicator of whether the machine is constant rpm or variable rpm. These parameters, defined more fully below, are required to compute the power coefficients A and B, if known values of A and B are not input to the program.

Figure 1 illustrates the model power output curve for an individual wind turbine in the array. The cut-in speed ( $V_{in}$ ), cutout speed ( $V_{out}$ ), rated speed ( $V_r$ ), and rated power ( $P_r$ ) definitions are illustrated schematically by this figure. The power curve between cut-in and rated speeds is characterized by a power coefficient model, which has been described more fully by Justus and Mikhail (1978b). The power coefficient ( $C_p$ ) is the ratio between the output power and the power available in the wind.  $C_p$  is modeled as illustrated in Figure 2 and, in general, is taken to reach a maximum value ( $C_{pm}$ ) at some

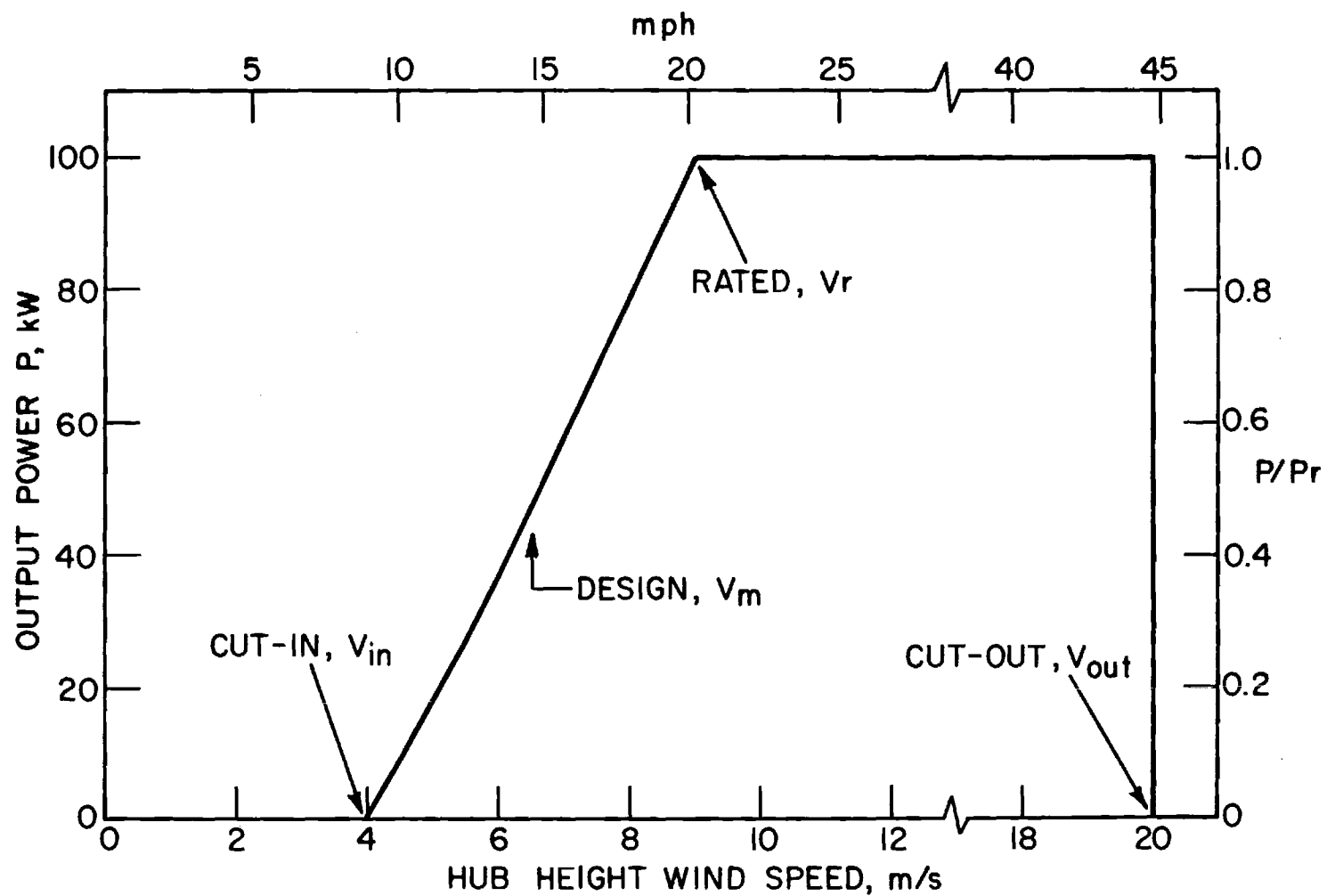


Figure 1. An Example Power Output Curve Versus Wind Speed

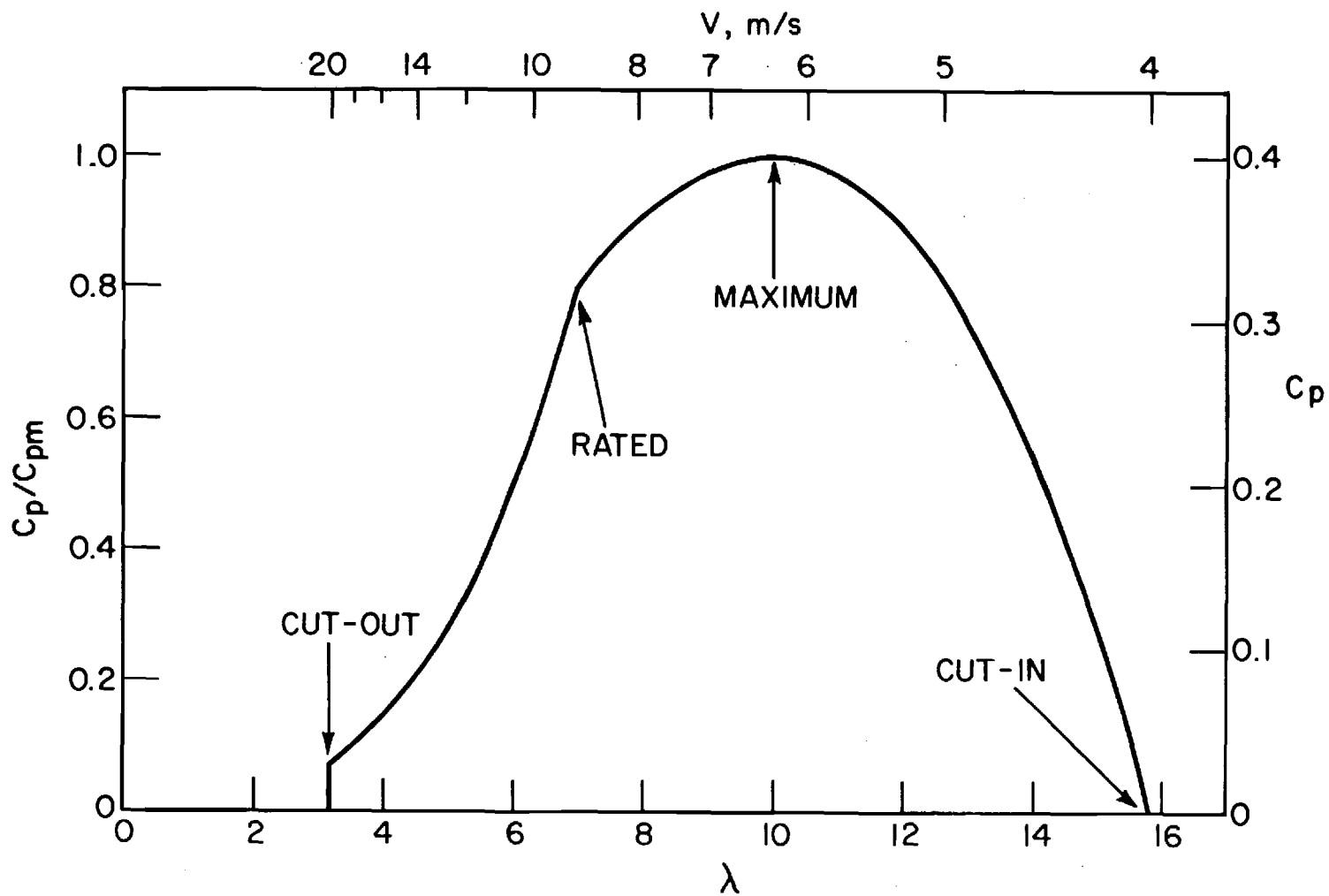


Figure 2. The Power Coefficient Curve Corresponding to Fig.1.  $C_p$  is Output Power Over Available Power in the Wind.  $\lambda$  is the Tip-Speed Ratio ( $R\Omega/V$ ).  $V$  is Hub-Height Wind Speed.

speed ( $V_m$ ) between cut-in and rated. The power coefficient at rated power is designated as  $C_{pr}$ . For variable rpm machines the value of the ratio  $C_{pr}/C_{pm}$  is assumed to be 1.0.  $C_{pr}/C_{pm} = 1.0$  is also assumed by the program if this ratio is unknown (input 0.0).

### Representative Site Input

Wind speed and other information from a single "representative" site must also be input to the program. As shown by Justus and Mikhail (1978a), the single representative site should be chosen first on the basis of appropriate mean wind speed. Secondly, the variance of winds at the representative site may be considered. If proper simulation of seasonal and diurnal shapes for array power is important, then the representative site should be selected as having, most nearly of all the available sites, seasonal and diurnal mean wind variations approximating those which would be obtained if averaged over all of the array site locations.

The normal mode of array simulation is based on wind speed input at the 10 m level, with height projection (done by the model) to hub height from the 10 m level. One parameter used for this height projection is the surface roughness ( $Z_0$ ) for the input site. Table 1 gives typical values for  $Z_0$  versus terrain and surface cover type. If  $Z_0$  is unknown (input value 0.0), the program assumes  $Z_0 = 0.05$  m.

The mean wind speed ( $\bar{V}_{10}$ ) at 10 m height, and some measure of variance of the wind speed, are also required by the model. If these parameters are already known at hub height, then they may be used as the input values, in which case hub height should be specified as 10 m in the input, to avoid any height projection by the program. The required measure of wind speed variance

Table 1. Typical Surface Roughnesses

<u>Type of Surface</u>	<u>Surface Roughness, <math>Z_0</math>, m</u>
Mud flats, ice	$10^{-5} - 3 \times 10^{-5}$
Smooth sea	$2 \times 10^{-4} - 3 \times 10^{-4}$
Sand	$10^{-4} - 10^{-3}$
Plain, snow covered	$4.9 \times 10^{-3}$
Grassy surface	0.017
Mown grass	$10^{-3} - 0.01$
Low grass, steppe	0.032
Flat country	0.021
High grass	0.039
Wheat	0.045
Beets	0.064
Palmetto	0.1 - 0.3
Low woods	0.05 - 0.1
High woods	0.2 - 0.9
Suburbia	1 - 2
City	1 - 4

can be input as either the Weibull shape factor ( $k_{10}$ ) at 10 m level, or as the standard deviation ( $\sigma_{10}$ ) at this height (or at hub height, if known, and hub height mean wind was input). The program uses the value for  $k_{10}$  if it is input. If  $\sigma_{10}$  is input, then  $k_{10}$  is calculated from (Justus and Mikhail, 1978b)

$$k_{10} = (\sigma_{10}/\bar{V}_{10})^{-1.086} . \quad (1)$$

If both  $k_{10}$  and  $\sigma_{10}$  are unknown (input as 0.0), then a value for  $k_{10}$  is computed from the reference wind statistics relation for median variance (Justus, 1978)

$$k_{10} = 0.94(\bar{V}_{10})^{\frac{1}{2}} . \quad (2)$$

The Weibull distribution scale factor at 10 m is calculated from the relation

$$c_{10} = \bar{V}_{10}/\Gamma(1 + 1/k_{10}) \quad (3)$$

where  $\Gamma$  is the usual gamma function.

The input values of speed and variance parameters must be for the time period for which array power statistics are to be evaluated. For example, if annual statistics are desired, then annual mean wind speed and variance parameters should be input. For monthly or seasonal statistics, the corresponding monthly or seasonal average and variance parameters should be input, etc.

#### Array Characteristics Input

The array of wind turbines to be modeled is illustrated schematically in Figure 3. The array is assumed to be made up of  $n$  separate sites. Each site may be a "farm" of wind turbines, with each farm, perhaps, having a different number of wind turbines. The numerical size of the array is characterized by



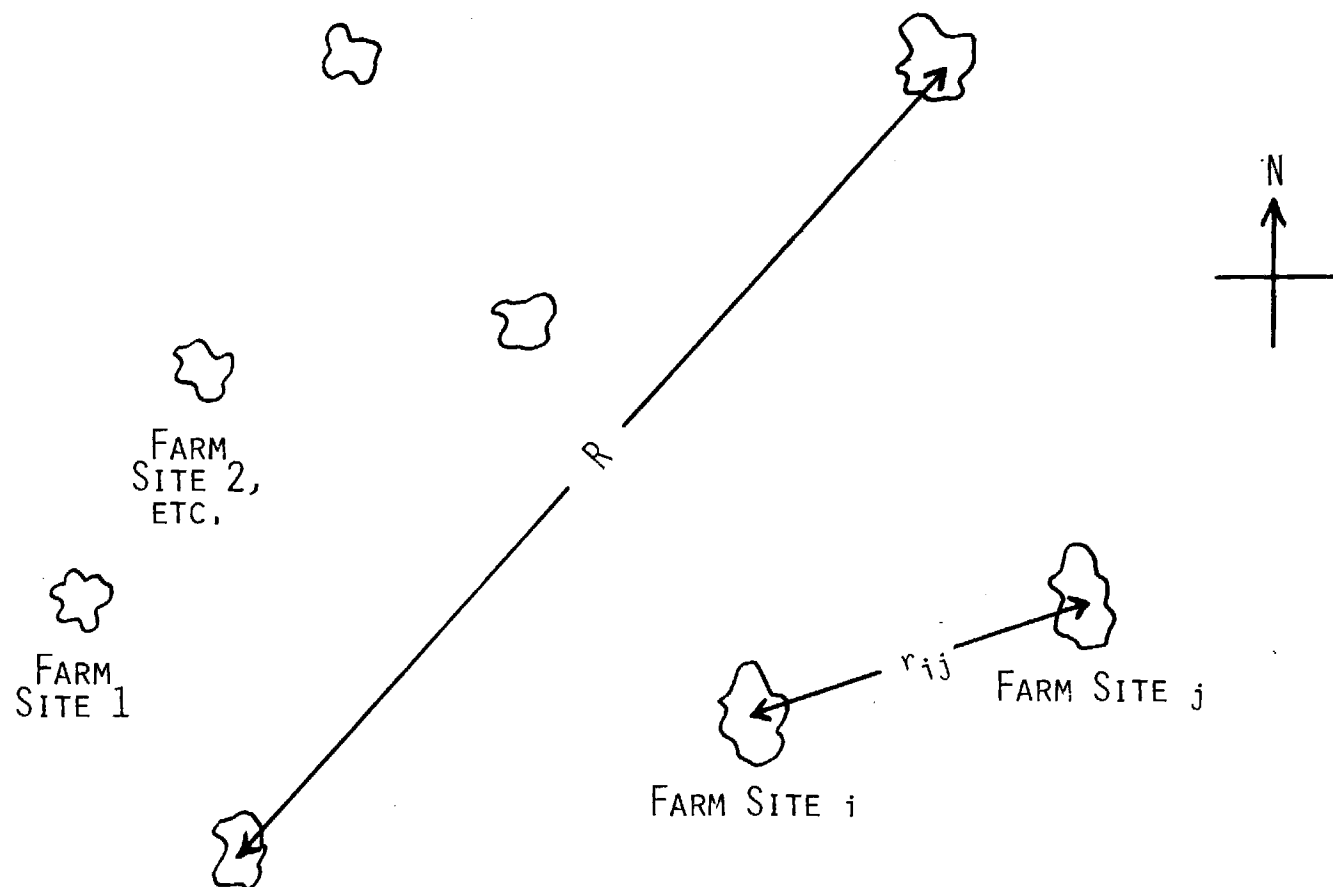


Figure 3. Schematic Map of an Array of  $n$  Sites (Farms) of Wind Turbines.  $R$  is Maximum Site Separation.  $\bar{r}$  is Average Over All  $i$  and  $j$  of Site Separations  $r_{ij}$ .

n, the number of array sites. The spatial size of the array is characterized by the maximum separation distance (R) between any of the site pairs in the array.

From these input parameters (n and R), the average site-pair separation ( $\bar{r}$ ) and the average spatial cross-correlation ( $\bar{\rho}$ ) are computed by the relations (Justus and Mikhail, 1978a)

$$\bar{r} = (n+1)R/[2.64(n-1)] \quad (4)$$

$$\bar{\rho} = \exp[-(\bar{r}/520)^{0.57}] \quad \bar{r} \geq 200 \text{ km} \quad (5)$$

$$\bar{\rho} = \exp[-(\bar{r}/1056)^{0.33}] \quad \bar{r} < 200 \text{ km.}$$

These parameters are used later to adjust between winds at the single input site to winds averaged across the array.

#### Power Regression Model Input

Based on the power curve model for an individual wind turbine, discussed above, the average output power ( $\bar{P}$ ) can be expressed as a linear function of the average wind speed ( $\bar{V}$ ), when normalized, respectively, by the rated power ( $P_r$ ) and rated speed ( $V_r$ ), i.e.

$$\bar{P}/P_r = A + B (\bar{V}/V_r), \quad 0 \leq \bar{P}/P_r \leq 1. \quad (6)$$

The methodology for evaluating these coefficients was documented by Justus and Mikhail (1978b). For the array model,  $\bar{V}$  is the mean wind speed averaged across the array, and  $\bar{P}$  is the array output power per wind turbine unit in the array (i.e. total output power from the array is  $\bar{P}$  times the number of wind turbine units in the array, not times n the number of sites in the array).

Values of the regression model power coefficients A and B may be input

directly into the program, or, if unknown, they are calculated from the equations

$$A = \alpha_0 + \alpha_1(V_{in}/V_r) + \alpha_5(C_{pr}/C_{pm}) \quad (7)$$

$$B = \alpha_2$$

where the  $\alpha$  coefficient values are built into the program, and were given by Justus and Mikhail (1978b). Values of  $V_{in}$ ,  $V_r$ , and the ratio  $C_{pr}/C_{pm}$  must be input to the program if equation (7) is employed. These parameters were discussed above.

#### The Height Projection Model

The Weibull parameters ( $C_{10}$  and  $k_{10}$ ) at 10 m height are adjusted to hub height ( $Z_h$ ) by the power law relation

$$C(Z_h) = C_{10} (Z_h/10)^\alpha, \quad (8)$$

and the equation

$$k(Z_h) = k_{10}/[1 - 0.088 \ln(Z_h/10)], \quad (9)$$

where the exponent ( $\alpha$ ) is given by

$$\alpha = [1/\ln(Z_g/10)] - 0.088 \ln(C_{10}/6), \quad (10)$$

and  $Z_g$  is the geometric mean height between 10 m and hub height, given by  $Z_g = \exp\{[\ln(10) + \ln(Z_h)]/2\}$ . This height projection methodology, including the dependence on surface roughness ( $Z_0$ ) was discussed by Justus (1978), and is a slight modification of the original equations presented by Justus and Mikhail (1976).

This height projection model was recently subjected to extensive study and comparison with observed height variations at Kennedy and Argonne towers, and compared to the theoretically-more-correct Monin-Obukov height projection method. These studies are being reported separately.

### Array Speed Distributions

Equations 8 and 9 are the Weibull parameters at hub height for the single representative input site. These are converted to the equivalent Weibull parameters for the average winds across the array, via the relations

$$\bar{V}_n = C(Z_h) \Gamma[1 + 1/k(Z_h)] \quad (11)$$

$$k_n = 0.893 k(Z_h) \{[1 + (n-1)\rho]/n\}^{-0.543} \quad (12)$$

$$C_n = \bar{V}_n / \Gamma(1 + 1/k_n). \quad (13)$$

The probability,  $p(V_i \leq V \leq V_j)$ , of observing winds ( $V$ ), at the single representative site, between any speed limits ( $V_i$  and  $V_j$ ) is found from the Weibull distribution with parameters  $C = C(Z_h)$  and  $k = k(Z_h)$ , from equations 8 and 9,

$$p(V_i \leq V \leq V_j) = \exp[-(V_i/c)^k] - \exp[-(V_j/c)^k]. \quad (14)$$

The probability,  $p(V_i \leq \bar{V} \leq V_j)$ , of observing winds ( $\bar{V}$ ), averaged across the array at any given time, between the speed limits  $V_i$  and  $V_j$ , is given by

$$p(V_i \leq \bar{V} \leq V_j) = \exp[-(V_i/c_n)^{k_n}] - \exp[-(V_j/c)^{k_n}] \quad (15)$$

where  $c_n$  and  $k_n$  are the Weibull parameters for the  $n$ -site array, from equations 12 and 13.

### Array Power Distributions

To find the probability,  $p(P_i \leq \bar{P} \leq P_j)$ , of observing array power ( $\bar{P}$ ) between any power limits ( $P_i$  and  $P_j$ ), the array power regression model, equation 6, is inverted to final equivalent speeds ( $V_i$  and  $V_j$ ) corresponding to the power limits, namely

$$\begin{aligned} V_i &= (P_i/P_r - A)(V_r/B) \\ V_j &= (P_j/P_r - A)(V_r/B). \end{aligned} \tag{16}$$

$P_r$  is rated power,  $V_r$  is rated speed,  $P_i$  and  $P_j$  are expressed as power per wind turbine unit, and coefficients  $A$  and  $B$  are evaluated from equation 7. After evaluating the equivalent speeds  $V_i$  and  $V_j$ , corresponding to powers  $P_i$  and  $P_j$ , equation 15 is also used to evaluate  $p(P_i \leq \bar{P} \leq P_j) = p(V_i \leq \bar{V} \leq V_j)$ .

### Time Series Winds and Power

In addition to computing the probability distributions, by equations 14 and 15, the program also evaluates time-series winds and powers from input time-series winds,  $V_{10}(t)$ , at the 10 m level, from the single representative site. First, the 10 m wind speed,  $V_{10}(t)$ , must be projected to a single-site wind speed  $V(Z_h, t)$  at hub height  $Z_h$ . The height projection method, equations 8 and 9, is based on the model that the 10 m wind at time  $t$  and the corresponding hub height wind at time  $t$  have the same cumulative probability, based on their respective probability distributions, i.e.

$$\begin{aligned} p(V_{10} \geq V_{10}(t)) &= \exp[-(V_{10}(t)/c_{10})^{k_{10}}] = p(V_{Z_h} \geq V(Z_h, t)) \\ &= \exp\{-[V(Z_h, t)/C(Z_h)]^{k(Z_h)}\} \end{aligned} \tag{17}$$

Equation (17) is solved for  $V(Z_h, t)$  to yield

$$V(Z_h, t) = c(Z_h) [V_{10}(t)/c_{10}]^{[k_{10}/k(Z_h)]}. \quad (18)$$

Height projection of time series winds by equation 18 is equivalent to height projection by a speed-dependent power law exponent  $\alpha$ , analogous to equations 8 and 10,

$$V(Z_h, t) = V_{10}(t)(Z_h/10)^\alpha \quad (19)$$

$$\alpha = [1/\ln(Z_g/10)] - 0.088 \ln(V_{10}/6).$$

Height projection of winds,  $V_{10}(t)$ , on a time-by-time basis by equation 18 or 19 produces a set of hub-height winds,  $V(Z_h, t)$ , which are distributed in a Weibull distribution, with hub-height  $c$  and  $k$  [ $c(Z_h)$  and  $k(Z_h)$ ] related to 10-meter  $c$  and  $k$  [ $c_{10}$  and  $k_{10}$ ] by equations 8 and 9.

After the 10-meter wind,  $V_{10}(t)$ , is converted to hub height,  $V(Z_h, t)$ , by equation (18), the equivalent array-average wind speed  $\bar{V}(t)$  is also found by probability distribution matching, analogous to equation 17,

$$\begin{aligned} p(\bar{V} \geq \bar{V}(t)) &= \exp\{-[\bar{V}(t)/c_n]^{k_n}\} = p(V_{Z_h} \geq V(Z_h, t)) \\ &= \exp\{-[V(Z_h, t)/c(Z_h)]^{k(Z_h)}\}. \end{aligned} \quad (20)$$

This equation is solved for  $\bar{V}(t)$  to yield

$$\bar{V}(t) = c_n [V(Z_h, t)/c(Z_h)]^{[k(Z_h)/k_n]} \quad (21)$$

Thus, the time series wind  $V_{10}(t)$  yields the equivalent hub height wind at the single site,  $V(Z_h, t)$ , by equation (18), and the equivalent array-average

wind speed  $\bar{V}(t)$ , by equation 21. The time series array power,  $\bar{P}(t)$ , per unit wind turbine, can be found from the power regression model and  $\bar{V}(t)$ , i.e.

$$\bar{P}(t) = P_r [A + B \bar{V}(t)/V_r] \quad (22)$$

The ability of this time series model to reproduce the time dynamics characteristics of actual arrays is documented in Appendix B.

### 3. SAMPLE PROGRAM OUTPUT

A listing of the array program is given in Appendix A. Table 2 gives a sample set of output of the program, with interactive input from a computer terminal. The wind turbine parameters used for input in the computer run shown in Table 2 are for the DOE/NASA Mod 0-A 200 kW wind turbine (NASA, 1978). Rated speed ( $V_r$ ) at hub height is 10 m/s. Rated power ( $P_r$ ) is 200 kW. The cut-in speed at hub height ( $V_{in}$ ) is 4.2 m/s, and the cutout speed at hub height ( $V_{out}$ ) is 17.9 m/s. The power coefficient ratio ( $C_{pr}/C_{pm}$ ) is 0.74, for this constant rpm machine. Hub height ( $Z_h$ ) is 30.5 m.

For the run shown in Table 2 the surface roughness ( $Z_0$ ) was entered as 0; the default value of 0.05 m was assumed by the program. The mean wind at 10 m level ( $\bar{V}_{10}$ ) was taken to be 7 m/s. Zero values of  $k_{10}$  and  $\sigma_{10}$  were input, so the program used the median reference statistics relations, equations 2 and 3, to calculate  $c_{10}$  and  $k_{10}$ . The output values shown in Table 2 are  $c_{10} = 7.89$  m/s and  $k_{10} = 2.49$ .

The array was taken to consist of 30 separate wind farms, with maximum site separation ( $R$ ) of 800 km. From these values, the average site separation ( $\bar{r}$ ) was found, by equation 4, to be 323.93 km. The average inter-site correlation ( $\bar{\rho}$ ) was found, from equation 5, to be 0.466.

Zero values were input for the wind power regression parameters ( $A$  and  $B$ ); hence, these factors were calculated from equation 7, with the  $\alpha$  values built into a program array, and with  $V_{in}/V_r$  and  $C_{pr}/C_{pm}$  from the program input.

Two values of wind speed at hub height (5 and 10 m/s) are entered as limits ( $V_i$  and  $V_j$ ) of wind speed intervals for the single-site and array distributions of wind speed,  $p(V_i \leq V \leq V_j)$  and  $p(V_i \leq \bar{V} \leq V_j)$ , of equations 14 and



Table 2. Sample Array Program Output, Interactive Input

```

***** ARRAY MODEL PROGRAM *****

ENTER RATED SPEED AT HUB HEIGHT, M/S, AND RATED POWER, KW
? 10. 200.
RATED SPEED, POWER =      10.0  200.0
ENTER SURFACE ROUGHNESS, M, OR 0.0 IF UNKNOWN
? 0.
SURFACE ROUGHNESS USED =      .050 M
ENTER MEAN SPEED AT 10 M, IN M/S
IF HUB HGT WIND IS KNOWN, ENTER IT AND USE HUB HGT = 10 M
? 7.
MEAN SPEED AT 10 M =      7.00 M/S
ENTER WEIBULL K FACTOR (OR 0.0) AND WIND SPEED SIGMA AT 10 M (OR 0.0)
IF BOTH ENTERED 0, STANDARD VALUES ARE ASSUMED
? 0. 0.
10 M WEIBULL C, K FACTORS =      7.89  2.49
ENTER HUB HGT, M, NUMBER OF ARRAY SITES, MAX SITE SEPARATION, KM, AND
POWER MODEL REGRESSION COEFFICIENTS A AND B OR 0.0'S IF UNKNOWN
? 30.5 30 800. 0. 0.
ENTER CUT-IN SPEED, CUT-OUT SPEED, M/S
? 4.2 17.9
CUT-IN, CUT-OUT SPEEDS =      4.2  17.9 M/S
ENTER 1 FOR CONSTANT RPM, 2 FOR VARIABLE RPM, OR 0 IF UNKNOWN
? 1
ENTER POWER COEFFICIENT RATIO, OR 0.0 IF UNKNOWN
? .74
CONSTANT RPM MACHINE WITH CPR/CPM =      .740
HUB HEIGHT =      30.50 M      N =      30
A,B =      -.273  1.030
R,RBAR,RHO =      800.00  323.93  .466
ENTER NUMBER OF WIND SPEEDS AND HUB HEIGHT SPEED VALUES, M/S,
FOR SINGLE SITE AND ARRAY DISTRIBUTIONS
NUMBER MUST BE GE 2 FOR OUTPUT
? 2 5. 10.
ENTER NUMBER AND VALUES OF POWERS, KW, FOR ARRAY POWER DISTRIBUTION
NUMBER MUST BE GE 2 FOR OUTPUT
? 2 0. 200.
POWER LAW EXPONENT =      .15
WIND SPEED, C1, K1, CN, KN AT HUB HEIGHT
      8.27  9.29  2.76  9.17  3.65
SPEEDS      PROB-1  PROB-N
0.0 - 5.0      16.6   10.3
5.0 - 10.0     54.0   64.3
10.0 - 99.9    29.4   25.4
POWERS      PROB-N
0.0 - 0.0      1.1
0.0 - 200.0    93.8
200.0 - 200.0  5.1
ENTER 10 M SINGLE SITE WIND SPEEDS, ONE AT A TIME, OR 99.9 TO STOP
SPEED-1 SPEED-1 SPEED-N POWER-N
10 M      HUB
? 4.
      4.0      5.0      5.8      64.3
? 8.
      8.0      9.4      9.3     136.1
? 12.
     12.0     13.6     12.2     196.7
? 99.9

```

15. Parameters for these distributions are: Weibull parameters,  $c(Z_h)$  and  $k(Z_h)$  for the single-site distribution at hub height, from equations 8 and 9, and the array Weibull parameters ( $c_n$  and  $k_n$ ) and array-average wind speed ( $\bar{V}_n$ ), from equations 11-13. The computed values shown in Table 2 are  $c(Z_h) = 9.29$  m/s,  $k(Z_h) = 2.76$ ,  $c_n = 9.17$  m/s,  $k_n = 3.65$ , and  $\bar{V}_n = 8.27$  m/s.

Two values of power per wind turbine (0 and 200 kW) are entered as limits ( $P_i$  and  $P_j$ ) of the power intervals for the array power distribution  $p(P_i \leq \bar{P} \leq P_j)$ .

The corresponding wind speeds,  $V_i$  and  $V_j$ , from these power limits are calculated, from equation 16, to be  $V_i = 2.65$  m/s and  $V_j = 12.36$  m/s. As discussed by Justus and Mikhail (1978a), these effective cut-in and cutout speeds for the array are, respectively, below and above the cut-in and cutout speeds for the individual wind turbines. Hence the array-power distribution has lower probability of being at zero power (or at full rated power) than is true for the power distribution for a single machine.

A sequence of single-site wind speeds at 10 m,  $V_{10}(t)$ , are input (values 4, 8, and 12 m/s in Table 2). The corresponding sequence of single-site wind speeds at hub height,  $V(Z_h, t)$ , are computed, from equation 19, to be 5.0, 9.4, and 13.6 m/s, respectively. The corresponding array-average wind speeds  $\bar{V}(t)$  are computed, from equation 21, to be 5.8, 9.3, and 12.2 m/s, respectively. Corresponding array power output values,  $\bar{P}(t)$ , per wind turbine unit, are computed, from equation 22, to be 64.3, 136.1, and 196.7 kW, respectively.

Table 3 shows sample output from the array program, with input coming from data on a computer file or on cards (batch input). The input values are the same as in Table 2, except that a larger set of speed limits ( $V_i$  and  $V_j$ ) and power limits ( $P_i$  and  $P_j$ ) are input for the wind speed and wind power distributions. Input values are "echoed" in the output shown in Table 3, so that the

user has a record of the input values which were input from the data file.

Figure 4 shows a plot of the wind speed distribution for the single site input and for the array. Values for the plot in Figure 4 are those shown in Table 3. Figure 4 illustrates the general nature of the transformation process from single-site to array wind speed distribution. The array distribution is always narrower and more sharply peaked about the average wind speed than is the single-site distribution. This feature, combined with the lower effective cut-in speed and higher effective cutout speed for the array, leads to higher probabilities of array power near the mean value, and lower probabilities of zero power (and full rated power).

Extensive testing of the array model, including studies of sensitivity of the model to variations in input parameters, has been conducted and documented by Justus and Mikhail (1978a). Testing of the time-series array simulation (equations 18, 21, and 22) is given in Appendix B.

Appendix C compares this array model program with that reported by Cliff et al. (1978), and derives one of the parameter values used in the Cliff model.

Table 3. Sample Array Program Output, Batch Input

```

***** ARRAY MODEL PROGRAM *****

ENTER RATED SPEED AT HUB HEIGHT, M/S, AND RATED POWER, KW
RATED SPEED, POWER = 10.0 200.0
ENTER SURFACE ROUGHNESS, M, OR 0.0 IF UNKNOWN
SURFACE ROUGHNESS USED = .050 M
ENTER MEAN SPEED AT 10 M, IN M/S
IF HUB HGT WIND IS KNOWN, ENTER IT AND USE HUB HGT = 10 M
MEAN SPEED AT 10 M = 7.00 M/S
ENTER WEIBULL K FACTOR (OR 0.0) AND WIND SPEED SIGMA AT 10 M (OR 0.0)
IF BOTH ENTERED 0, STANDARD VALUES ARE ASSUMED
10 M WEIBULL C, K FACTORS = 7.89 2.49
ENTER HUB HGT, M, NUMBER OF ARRAY SITES, MAX SITE SEPARATION, KM, AND
POWER MODEL REGRESSION COEFFICIENTS A AND B OR 0.0'S IF UNKNOWN
ENTER CUT-IN SPEED, CUT-OUT SPEED, M/S
CUT-IN, CUT-OUT SPEEDS = 4.2 17.9 M/S
ENTER 1 FOR CONSTANT RPM, 2 FOR VARIABLE RPM, OR 0 IF UNKNOWN
ENTER POWER COEFFICIENT RATIO, OR 0.0 IF UNKNOWN
CONSTANT RPM MACHINE WITH CPR/CPM = .740
HUB HEIGHT = 30.50 M N = 30
A, B = -.273 1.030
R, RBAR, RHO = 800.00 323.93 .466
ENTER NUMBER OF WIND SPEEDS AND HUB HEIGHT SPEED VALUES, M/S,
FOR SINGLE SITE AND ARRAY DISTRIBUTIONS
NUMBER MUST BE GE 2 FOR OUTPUT
ENTER NUMBER AND VALUES OF POWERS, KW, FOR ARRAY POWER DISTRIBUTION
NUMBER MUST BE GE 2 FOR OUTPUT
POWER LAW EXPONENT = .15
WIND SPEED, C1, K1, CN, KN AT HUB HEIGHT
8.27 9.29 2.76 9.17 3.65
SPEEDS PROB-1 PROB-N
0.0 - 2.0 1.4 .4
2.0 - 4.0 7.9 4.3
4.0 - 6.0 16.5 14.4
6.0 - 8.0 22.5 26.4
8.0 - 10.0 22.2 29.1
10.0 - 12.0 16.2 18.4
12.0 - 14.0 8.7 6.0
14.0 - 16.0 3.4 .9
16.0 - 99.9 1.1 .0
POWERS PROB-N
0.0 - 0.0 1.1
0.0 - 50.0 9.8
50.0 - 100.0 27.3
100.0 - 150.0 35.6
150.0 - 200.0 21.1
200.0 - 200.0 5.1
ENTER 10 M SINGLE SITE WIND SPEEDS, ONE AT A TIME, OR 99.9 TO STOP
SPEED-1 SPEED-N
10 M HUB
3.0 3.9 4.7 43.1
4.0 5.0 5.8 64.3
5.0 6.2 6.7 83.8
6.0 7.3 7.6 102.1
7.0 8.3 8.5 119.5
8.0 9.4 9.3 136.1
9.0 10.5 10.0 152.0
10.0 11.5 10.8 167.4
11.0 12.5 11.5 182.2

```

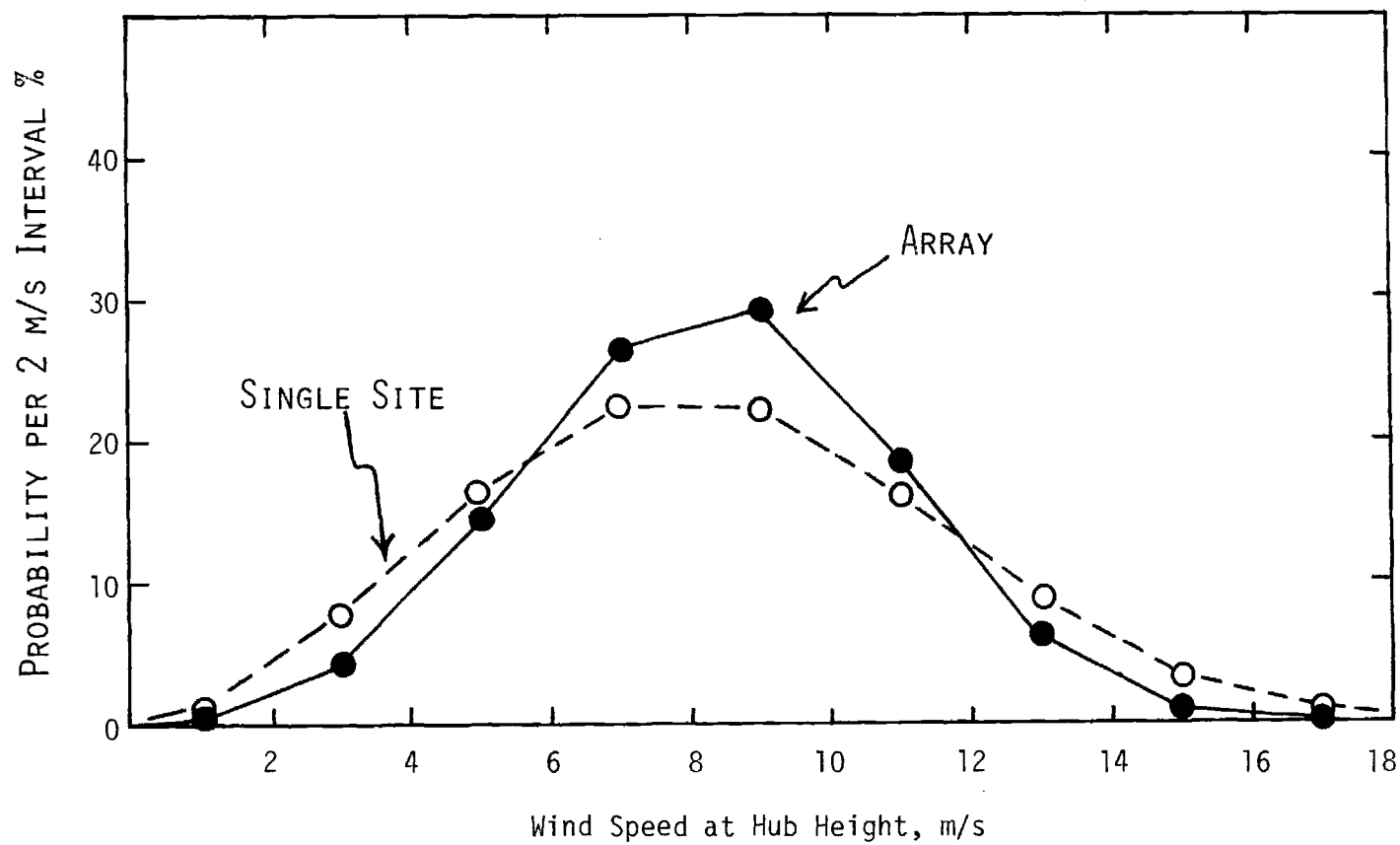


Figure 4. Sample Wind Speed Distribution Output,  
(from data in Table 3).

## REFERENCES

- Cliff, W. C., C. G. Justus, and C. E. Elderkin (1978): "Simulation of the Hourly Wind Speeds for Randomly Dispersed Sites", PNL-2523.
- Justus, C. G. (1976): "Wind Energy Statistics for Large Arrays of Wind Turbines (New England and Central U. S. Regions)", ERDA/NSF-00547/76/1 (NSF/RA-76-0191).
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- Justus, C. G. and Amir Mikhail (1978b): "Generic Power Performance Estimates for Wind Turbines", Wind Technology Journal, 2, 45-62.
- NASA (1978): "200 Kilowatt Wind Turbine Project", January.

## APPENDIX A

### ARRAY MODEL PROGRAM LISTING

The following is a computer listing of the array model program. All computational steps are described by the equations given in Section 2. The subroutine GAM(XI) is a polynomial approximation for the gamma function  $\Gamma(x)$ , used in converting Weibull  $c$  and  $k$  to mean wind speed  $\bar{V}$ , as in equations 3 and 11.



```

PROGRAM ARRAY(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
  DIMENSION V(20),P(20),VP(20),A0(24),A1(24),A2(24),A5(24),
  $ TYPE(2)
  REAL K10,KN,KZ,KL,KM,KH
  INTEGER TYPE
C.....A0,A1,A2,A5 = POWER MODEL USED TO COMPUTE A AND B OF
C.....P/PR = A+B*(V/VR)
  DATA TYPE /"CONSTANT","VARIABLE"/
  DATA A0 / .427,.409,.358,-.035,.019,.142,-.107,-.074,.016,
  $ -.109,-.078,-.008,.190,.187,.173,-.273,-.203,-.044,-.345,
  $ -.295,-.170,-.346,-.300,-.193 /
  DATA A1 / -.190,-.182,-.161,-.190,-.182,-.161,-.190,-.182,-.161,
  $ -.190,-.182,-.161,-.106,-.104,-.099,-.106,-.104,-.099,
  $ -.105,-.104,-.099,-.106,-.104,-.099 /
  DATA A2 / .175,.152,.106,1.054,.941,.660,1.170,1.095,.888,
  $ 1.172,1.192,.927,.147,.129,.091,1.026,.918,.645,1.142,
  $ 1.072,.874,1.145,1.078,.913 /
  DATA A5 / -.236,-.219,-.181,-.236,-.219,-.181,-.236,-.219,
  $ -.181,-.236,-.219,-.181,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0.,0./
  WRITE(6,80)
80  FORMAT("1")
  WRITE(6,85)
85  FORMAT(14X,"***** ARRAY MODEL PROGRAM *****",/)
  WRITE(6,90)
90  FORMAT(" ENTER RATED SPEED AT HUB HEIGHT, M/S, AND RATED ",
  $ "POWER, KW")
  READ(5,*) VR,PR
  WRITE(6,100) VR,PR
100 FORMAT(" RATED SPEED, POWER = ",2F7.1)
  WRITE(6,110)
110 FORMAT(" ENTER SURFACE ROUGHNESS, M, OR 0.0 IF UNKNOWN")
  READ(5,*) Z0
  IF (Z0.LE.0.0) Z0 = 0.05
  WRITE(6,120) Z0
120 FORMAT(" SURFACE ROUGHNESS USED = ",F6.3," M")
  WRITE(6,130)
130 FORMAT(" ENTER MEAN SPEED AT 10 M, IN M/S")
  WRITE(6,135)
135 FORMAT(" IF HUB HGT WIND IS KNOWN, ENTER IT AND USE HUB "
  $ ," HGT = 10 M")
  READ(5,*) V10
  WRITE(6,140) V10
140 FORMAT(" MEAN SPEED AT 10 M = ",F6.2," M/S")
  WRITE(6,150)
150 FORMAT(" ENTER WEIBULL K FACTOR (OR 0.0) AND WIND SPEED",
  $ " SIGMA AT 10 M (OR 0.0)",/, " IF BOTH ENTERED 0, ",
  $ "STANDARD VALUES ARE ASSUMED")
  READ(5,*) K10,S10
C.....ENTER EITHER 10M WEIBULL SHAPE FACTOR (K) VALUE (IF KNOWN)
C.....OR STANDARD DEVIATION (IF KNOWN). IF K VALUE IS ENTERED,
C.....IT IS USED. IF BOTH ARE ENTERED AS 0,0 (UNKNOWN), THEN
C.....THE AVERAGE REFERENCE STATISTICS MODEL IS USED TO CALCULATE
C.....K FROM THE MEAN WIND SPEED.
  IF (K10.LE.0.) GO TO 160
  GO TO 190
160 IF(S10.LE.0.) GO TO 170
  GO TO 180

```

```

170 K10 = 0.94*SQRT(V10)
    GO TO 190
180 K10 = (S10/V10)**(-1.086)
190 CONTINUE
    C10 = V10/GAM(1. + 1./K10)
    WRITE(6,200) C10,K10
200 FORMAT(" 10 M WEIBULL C, K FACTORS = ",2F6.2)
    WRITE(6,210)
210 FORMAT(" ENTER HUB HGT, M, NUMBER OF ARRAY SITES,"
$ " MAX SITE SEPARATION, KM, AND ",/,
$ " POWER MODEL REGRESSION COEFFICIENTS A AND B",
$ " OF 0.0'S IF UNKNOWN")
C..... A AND B ARE POWER CO-EFFICIENTS FOR P/PR = A+B*(V/VR)
    READ(5,*) Z,N,R,A,B
C..... ZG = GEOMETRIC MEAN HEIGHT BETWEEN 10M AND HUB HEIGHT.
    ZG = EXP((ALOG(10.) + ALOG(Z))/2.)
    IF (B.GT.0.) GO TO 2195
    WRITE(6,2110)
2110 FORMAT(" ENTER CUT-IN SPEED, CUT-OUT SPEED, M/S")
C..... CUT-IN AND CUT-OFF SPEEDS, NEEDED ONLY IF POWER A AND B
C..... ARE NOT KNOWN.
    READ(5,*) VIN,VOUT
    WRITE(6,2120) VIN,VOUT
    VIN = VIN/VR
    IF (VIN.LT.0.3) VIN = .3
    IF (VIN.GT.0.8) VIN = .8
    VOUT = VOUT/VR
    IF (VOUT.LT.1.) VOUT = 1.
    IF (VOUT.GT.2.5) VOUT = 2.5
C..... VIN/VR RESTRICTED TO MEANINGFUL RANGE FOR POWER CO-EFFICIENT MODEL
2120 FORMAT(" CUT-IN, CUT-OUT SPEEDS = ",2F6.1," M/S")
    WRITE(6,2130)
2130 FORMAT(" ENTER 1 FOR CONSTANT RPM, 2 FOR VARIABLE RPM,"
$ " OR 0 IF UNKNOWN")
    READ(5,*) IRPM
    IF (IRPM.LE.0) IRPM = 1
    IF (IRPM.GE.2) IRPM = 2
    WRITE(6,2140)
2140 FORMAT(" ENTER POWER COEFFICIENT RATIO, OR 0.0 IF UNKNOWN")
C..... CPR/CPM = RATIO OF POWER CO-EFFICIENT AT RATED AND AT MAXIMUM
C..... CP CONDITIONS. CPR/CPM = 1 IS ASSUMED FOR VARIABLE RPM MACHINES
C..... AND IF UNKNOWN.
    READ(5,*) CPRCPM
    IF (IRPM.EQ.2) CPRCPM = 1.
    IF (CPRCPM.LE.0.) CPRCPM = 1.
    IF (CPRCPM.GT.1.) CPRCPM = 1.
    WRITE(6,2150) TYPE(IRPM),CPRCPM
2150 FORMAT(1X,A8," RPM MACHINE WITH CPR/CPM = ",F6.3)
    XV = 1.
    IV = 1
    IF (VOUT.LT.1.5) GO TO 2170
    IV = 4
    XV = 1.5
    IF (VOUT.LT.2.0) GO TO 2170
    IV = 7
    XV = 2.0
2170 IF (IRPM.EQ.2) IV = IV + 12

```

```

VFAC = (VOUT - XV)/.5
C.....INTERPOLATED POWER CO-EFFICIENTS FOR LOW, MEDIAN AND HIGH
C.....REFERENCE STATISTICS CASES.
  A0L = A0(IV) + (A0(IV+3) - A0(IV))*VFAC
  A0M = A0(IV+1) + (A0(IV+4) - A0(IV+1))*VFAC
  A0H = A0(IV+2) + (A0(IV+5) - A0(IV+2))*VFAC
  A1L = A1(IV) + (A1(IV+3) - A1(IV))*VFAC
  A1M = A1(IV+1) + (A1(IV+4) - A1(IV+1))*VFAC
  A1H = A1(IV+2) + (A1(IV+5) - A1(IV+2))*VFAC
  A2L = A2(IV) + (A2(IV+3) - A2(IV))*VFAC
  A2M = A2(IV+1) + (A2(IV+4) - A2(IV+1))*VFAC
  A2H = A2(IV+2) + (A2(IV+5) - A2(IV+2))*VFAC
  A5L = A5(IV) + (A5(IV+3) - A5(IV))*VFAC
  A5M = A5(IV+1) + (A5(IV+4) - A5(IV+1))*VFAC
  A5H = A5(IV+2) + (A5(IV+5) - A5(IV+2))*VFAC
  SRV = SQRT(V10)
C.....FINDS WEIBULL K VALUE CORRESPONDING TO LOW, MEDIAN AND HIGH
C.....REFERENCE DISTRIBUTION CASES.
  KL = 1.05*SRV
  KM = .94*SRV
  KH = .73*SRV
C.....INTERPOLATE TO FIND POWER COEFFICIENTS BASED ON ACTUAL K BEING
C.....BETWEEN LOW AND MEDIAN OR MEDIAN AND HIGH REFERENCES STATISTICS
  IF (K10.GE,KM) GO TO 2180
  VFAC = (K10 - KH)/(KM - KH)
  FA0 = A0H + (A0M - A0H)*VFAC
  FA1 = A1H + (A1M - A1H)*VFAC
  FA2 = A2H + (A2M - A2H)*VFAC
  FA5 = A5H + (A5M - A5H)*VFAC
  GO TO 2190
2180 VFAC = (K10 - KM)/(KL - KM)
  FA0 = A0M + (A0L - A0M)*VFAC
  FA1 = A1M + (A1L - A1M)*VFAC
  FA2 = A2M + (A2L - A2M)*VFAC
  FA5 = A5M + (A5L - A5M)*VFAC
2190 A = FA0 + FA1*VIN + FA5*CPRCPM
  B = FA2
C.....A AND B = POWER COEFFICIENTS FOR POWER MODEL P/PR = A+B*(V/VR)
C.....RBAR = AVERAGE SEPARATION DISTANCE BETWEEN ARRAY SITE PAIRS.
2195 RBAR = (N + 1.)*R/(2.64*(N - 1.))
  IF (RBAR.GE.200.) RHO = EXP(-((RBAR/520.)**.57))
  IF (RBAR.LT.200.) RHO = EXP(-((RBAR/1056.)**.33))
  WRITE(6,220)Z,N,A,B,R,RBAR,RHO
220 FORMAT(" HUB HEIGHT = ",F8.2," M ",3X,"N = ",I4,/,
  $ " A,B = ",2F8.3,/, " R,RBAR,RHO = ",2F8.2,F7.3)
  WRITE(6,230)
C.....LOWEST SPEED ASSUMED 0.0, HIGHEST SPEED ASSUMED 99.9. THESE
C.....VALUES NEED NOT BE ENTERED.
230 FORMAT(" ENTER NUMBER OF WIND SPEEDS AND HUB HEIGHT SPEED VALUES
  $ " M/S,/, " FOR SINGLE SITE AND ARRAY DISTRIBUTIONS",/,
  $ " NUMBER MUST BE GE 2 FOR OUTPUT")
  READ(5,*)NV,(V(I),I=1,NV)
  WRITE(6,240)
C.....ENTER POWERS OF 0.0 AND RATED IF PROBABILITIES OF FINDING THESE
C.....POWERS ARE DESIRED.
240 FORMAT(" ENTER NUMBER AND VALUES OF POWERS, KW,"
  $ " FOR ARRAY POWER DISTRIBUTION",/,

```

```

$ " NUMBER MUST BE GE 2 FOR OUTPUT")
READ(5,*) NP, (P(I),I=1,NP)
C.....ALPH = POWER LAW HEIGHT VARIATION EXPONENT.
      ALPH = (1./ALOG(ZG/Z0)) - 0.088*ALOG(C10/6.)
C.....CZ,KZ = WEIBULL PARAMETERS AT HUB HEIGHT FOR SINGLE SITE.
C.....CN,KN = WEIBULL PARAMETERS AT HUB HEIGHT FOR ARRAY.
C.....VN = ARRAY MEAN WIND SPEED AT HUB HEIGHT.
      CZ = C10*((Z/10.)**ALPH)
      KZ = K10/(1. - 0.088*ALOG(Z/10.))
      VN = CZ*GAM(1. + 1./KZ)
      KN = KZ*.893*((1.+(N-1.)*RHO)/N)**(-.543)
      CN = VN/GAM(1. + 1./KN)
      WRITE(6,245)ALPH
245  FORMAT(" POWER LAW EXPONENT =",F6.2)
      WRITE(6,250) VN,CZ,KZ,CN,KN
250  FORMAT(" WIND SPEED,   C1,   K1,   CN,   KN AT HUB HEIGHT",
$ /,5X,5F6.2)
      IF (NV.LT.2) GO TO 285
      P1I = (1. - EXP(-(V(1)/CZ)**KZ))*100.
      PNI = (1. - EXP(-(V(1)/CN)**KN))*100.
      WRITE(6,260)
C.....PROBABILITY P1I OF SINGLE SITE WINDS AT HUB HEIGHT IN INTERVAL
C.....V(I-1) TO V(I).  PROBABILITY PNI OF ARRAY AVERAGE WIND IN
C.....INTERVAL V(I-1) TO V(I).
260  FORMAT("  SPEEDS      PROB-1  PROB-N")
      WRITE(6,270)0.,V(1),P1I,PNI
270  FORMAT(1X,F4.1," - ",F4.1,2F7.1)
      DO 280 I = 2,NV
      P1I = (EXP(-(V(I-1)/CZ)**KZ) - EXP(-(V(I)/CZ)**KZ))*100.
      PNI = (EXP(-(V(I-1)/CN)**KN) - EXP(-(V(I)/CN)**KN))*100.
      WRITE(6,270) V(I-1),V(I),P1I,PNI
280  CONTINUE
      P1I = EXP(-(V(NV)/CZ)**KZ)*100.
      PNI = EXP(-(V(NV)/CN)**KN)*100.
      WRITE(6,270) V(NV),99.9,P1I,PNI
285  IF (NP.LT.2) GO TO 325
      DO 290 I = 1,NP
290  VP(I) = (P(I)/PR - A)*VR/B
      PNI = (1. - EXP(-(VP(I)/CN)**KN))*100.
      WRITE(6,300)
300  FORMAT(5X,"POWERS",6X,"PROB-N")
      WRITE(6,310) 0., P(1),PNI
C.....PROBABILITIES OF ARRAY POWER IN INTERVAL P(I-1) TO P(I)
310  FORMAT(1X,F6.1," - ",F6.1,F7.1)
      DO 320 I = 2,NP
      PNI = (EXP(-(VP(I-1)/CN)**KN) - EXP(-(VP(I)/CN)**KN))*100.
      WRITE(6,310) P(I-1),P(I),PNI
320  CONTINUE
      PNI = EXP(-(VP(NP)/CN)**KN)*100.
      WRITE(6,310) P(NP),PR,PNI
325  WRITE(6,330)
330  FORMAT(" ENTER 10 M SINGLE SITE WIND SPEEDS, ONE AT A TIME, OR",
$ " 99.9 TO STOP",/, " SPEED-1 SPEED-1 SPEED-N POWER-N",/,
$ " 10 M",5X,"HUB")
340  READ(5,*) V1
      IF (V1.GE.99.899) STOP
C.....V1 = INPUT 10M SINGLE SITE WIND

```

```

C.....VD = HUB HEIGHT SINGLE SITE WIND BY DISTRIBUTION MATCHING
C.....VN = ARRAY WIND AT HUB HEIGHT.
      VD = CZ*((V1/C10)**(K10/KZ))
      VN = CN*((VD/CZ)**(KZ/KN))
      PN = (A + B*(VN/VR))*PR
      IF (PN.LT.0.) PN = 0.
      IF (PN.GT.PR) PN = PR
      WRITE(6,350) V1,VD,VN,PN
350 FORMAT(4F8.1)
      GO TO 340
      END

```

```

      FUNCTION GAM(XI)
      DIMENSION A(8)
      DATA A / -0.57719155, 0.98820589, -0.89705694, 0.91820686,
$          -0.75670408, 0.48219939, -0.19352782, 0.03586834 /
      NM = 0
      NP = 0
      IF (XI.LT.1.) GO TO 10
      IF (XI.GT.2.) GO TO 20
      X = XI
      GO TO 50
10  CONTINUE
      NM = INT(2 - XI)
      X = XI + NM
      GO TO 50
20  CONTINUE
      NM = INT(XI - 1)
      X = XI - NM
50  CONTINUE
      XF = X - 1
      GOUT = A(8)*XF
      DO 60 II = 1,7
      I = 8 - II
      GOUT = (GOUT + A(I))*XF
60  CONTINUE
      GOUT = GOUT + 1.
      IF (NM .LT. 1) GO TO 80
      DO 70 J = 1,NM
      GOUT = GOUT/(X - J)
70  CONTINUE
      GO TO 100
80  CONTINUE
      IF (NP.LT.1) GO TO 100
      DO 90 J = 1,NP
      GOUT = GOUT*(X + J - 1)
90  CONTINUE
100 CONTINUE
      GAM = GOUT
      RETURN
      END

```

## APPENDIX B

### THE TIME-SERIES ARRAY MODEL

The transformation between time-series wind speed at a single site,  $V(Z_h, t)$ , and the corresponding array-average wind speed,  $\bar{V}(t)$ , is based on equating the cumulative probabilities from their respective distributions, as in equation 20. This method insures that the time-series array-average wind speeds,  $\bar{V}(t)$ , will have the appropriate probability distribution. However, it is necessary to test the validity of the model results for the time dynamics of  $\bar{V}(t)$ , or better yet, the time dynamics of the corresponding array power  $\bar{P}(t)$ , evaluated by equation 22.

In order to test the validity of the time dynamics of the  $\bar{P}(t)$  values generated by this model, probability distributions  $p(|\bar{P}(t + \Delta t) - \bar{P}(t)| \leq \Delta P)$  were evaluated for various magnitudes of output power change,  $\Delta P$ , and for various time changes  $\Delta t$ . Figures B-1 through B-3 show these results for the Northeast array (Boston, Blue Hill, Providence, Laguardia, and Atlantic City). Values plotted as open circles are the probabilities for power change  $\Delta P$  from an average individual wind turbine in this array; x's are the actual observed array power change probabilities; and dots are the model array power change probabilities, evaluated from time-series array-average winds computed by equations 21 and 22. Power output simulations in Figures B-1 through B-3 are for the 2 MW rated power wind turbine previously studied in the array analysis by Justus and Mikhail (1978a). In all three Figures, power-change probabilities for the model array agree much closer to those for the observed array than those for an individual wind turbine. It is concluded that the time-series model adequately simulates the time dynamics of array-average winds and wind power output.

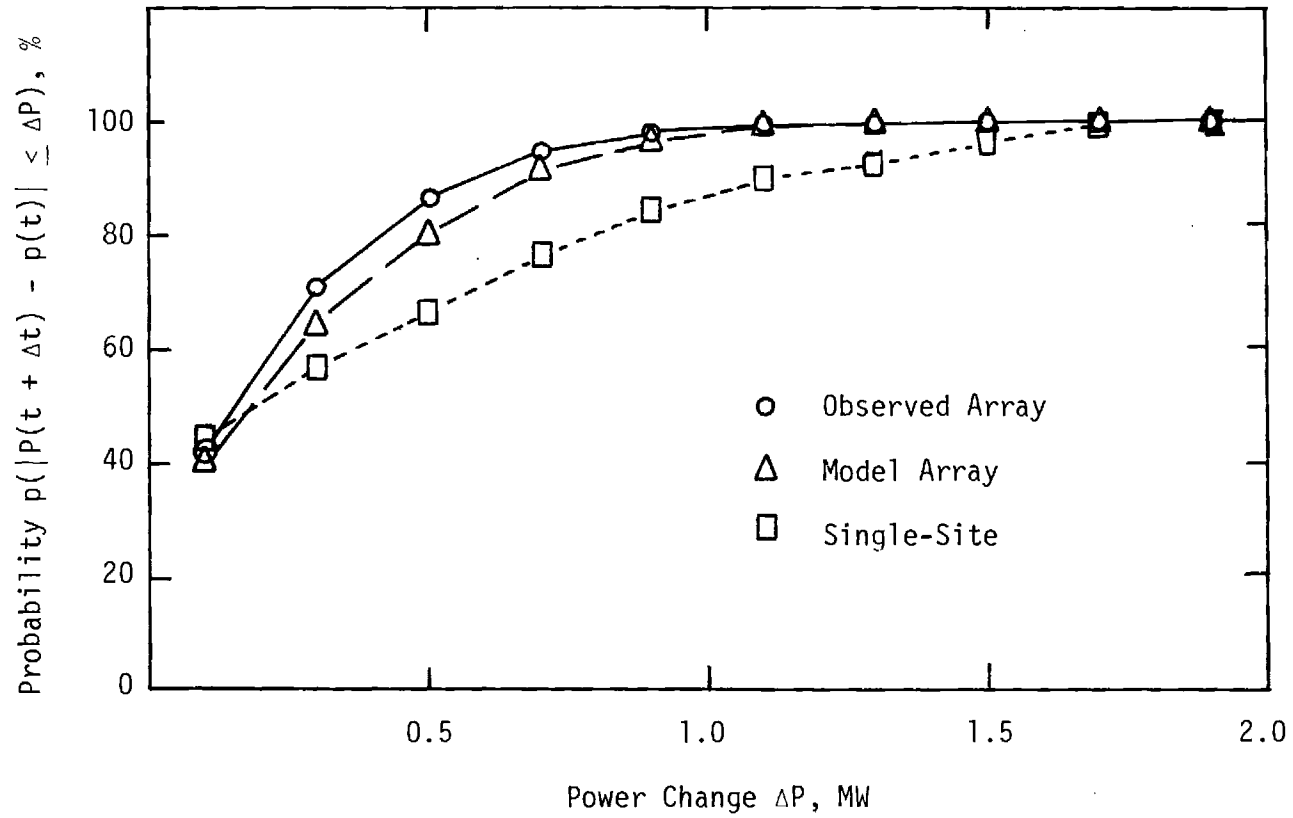


Figure B-1. Probability  $p(|P(t + \Delta t) - P(t)| \leq \Delta P)$  for Time Change  $\Delta t = 3$  Hours, Northeast Array



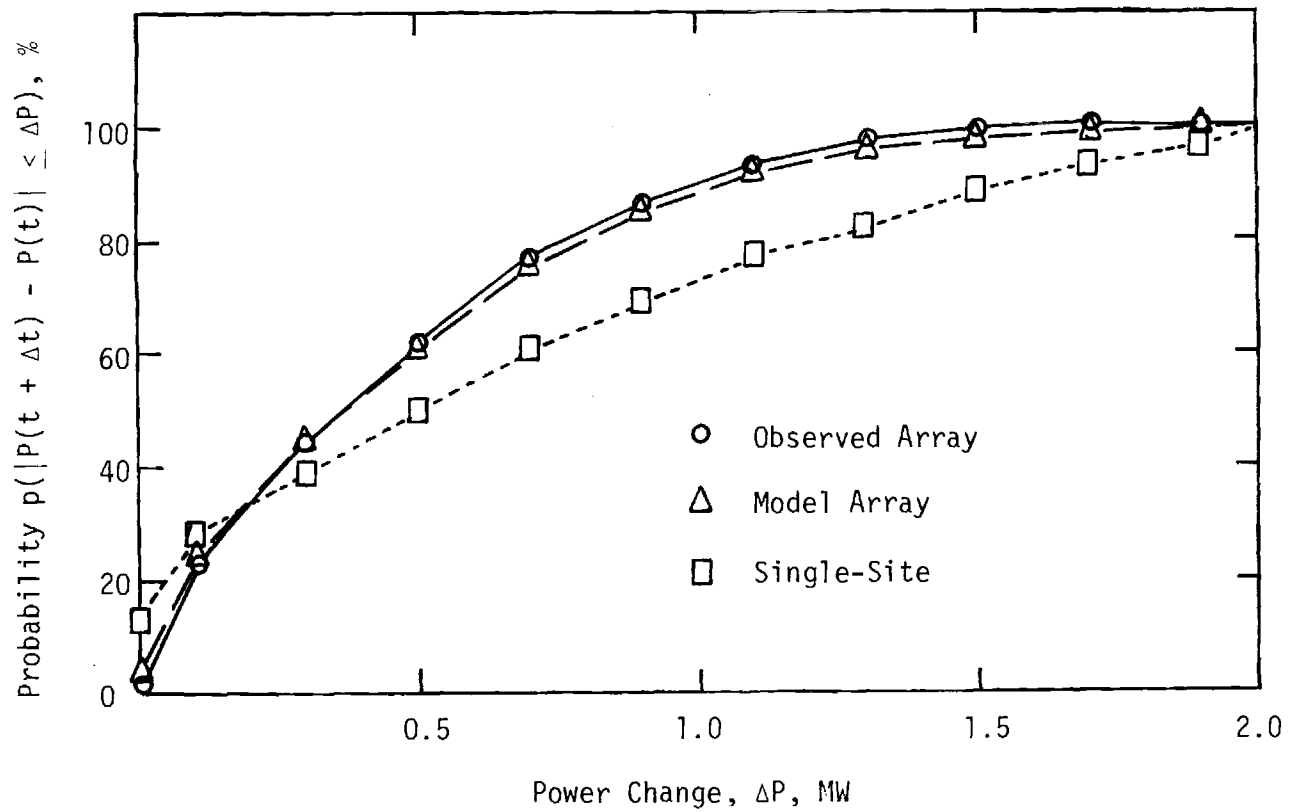


Figure B-2. Probability  $p(|P(t + \Delta t) - P(t)| \leq \Delta P)$  for Time Change  $\Delta t = 12$  Hours, Northeast Array

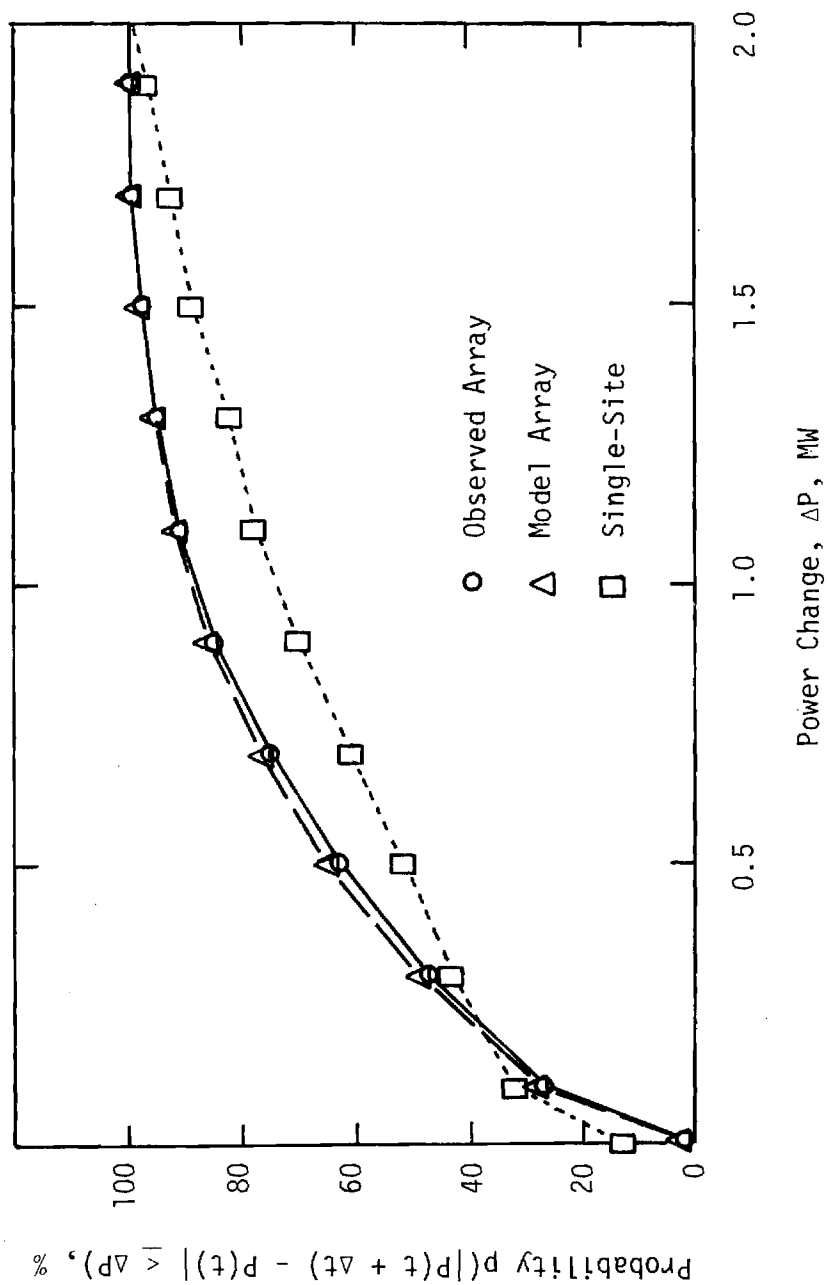


Figure B-3. Probability  $p(|P(t + \Delta t) - P(t)| \leq \Delta P)$  for Time Change  $\Delta t = 24$  Hours, Northeast Array

## APPENDIX C

### COMPARISON WITH CLIFF ET AL. ARRAY MODEL

The array model presented here is based on the transformation of the Weibull distribution for single-site winds to a narrower (higher  $k$  value) Weibull distribution of array-average wind speed,  $p(\bar{V})$ . Power output from the array is modeled as a linear function of array average wind speed, given by equation 6. Another array model, reported by Cliff, Justus, and Elderkin (1978) is based on the same transformation from single-site to array-average wind speed. However, the model by Cliff et al. also evaluates a variance,  $\sigma_a^2$ , of wind speeds about the array-average wind speed,  $\bar{V}$ , and uses this as the basis to simulate time series winds at any of the individual sites within the array from the time-series winds at the "representative" input site. Thus, in the model of Cliff, et al., input values of single-site winds,  $V(t)$ , are converted to array average winds,  $\bar{V}(t)$ , and variances,  $\sigma_a^2(t)$ . Time series winds at any site  $i$ ,  $V_i(t)$ , are then simulated by a random selection from the distribution across the array, characterized at time  $t$  by  $\bar{V}(t)$  and  $\sigma_a^2(t)$ . This distribution across the array is also characterized as a Weibull distribution with shape factor given by

$$k_a = (\sigma_a/\bar{V})^{-1.086}. \quad (C-1)$$

The conversion between the shape factor,  $k_s$ , for the single-site distribution and  $k_a$  for the distribution across the array, was modeled by Cliff et al. as

$$k_a = k_s / (1 - \bar{\rho})^C \quad (C-2)$$

where  $\bar{\rho}$  is the average spatial cross-correlation for the array, given, for example, by equation 5 of this report.

For relations C-1 and C-2 to be valid, and consistent with the median value reference statistics relation between  $k_s$  and  $\bar{V}$  (c.f. equation 2),  $k_s$  must satisfy the relation

$$k_s = 0.94 \bar{V}^{-1/2}. \quad (C-3)$$

Then  $\sigma_a$  and  $\bar{V}$  must be related by

$$\begin{aligned} \sigma_a &= [0.94/(1 - \bar{\rho})^c]^{-1/1.086} \bar{V}^{0.54} \\ &= 1.054 (1 - \bar{\rho})^{0.921c} \bar{V}^{0.54}. \end{aligned} \quad (C-4)$$

This equation shows that the dependence of  $\sigma_a$  on  $\bar{V}$  varies with correlation ( $\bar{\rho}$ ) in a way which depends on the value of the parameter  $c$ . Figure C-1 shows a plot of the theoretical ratio  $\sigma_a/(\bar{V})^{0.54} = 1.054 (1 - \bar{\rho})^{0.921c}$ , versus correlation ( $\bar{\rho}$ ) and various assumed values for the parameter  $c$ . Solid dots in Figure C-1 are observed slopes for four plots of  $\sigma_a$  versus  $(\bar{V})^{0.54}$ , from four arrays previously studied (Justus and Mikhail, 1978a). Although there is some scatter at the low  $\bar{\rho}$  values, Figure C-1 indicates that  $c = 0.5$  is an appropriate value to use in equation C-2. This is the value that was used by Cliff et al., although justification for this value was not presented in their report.

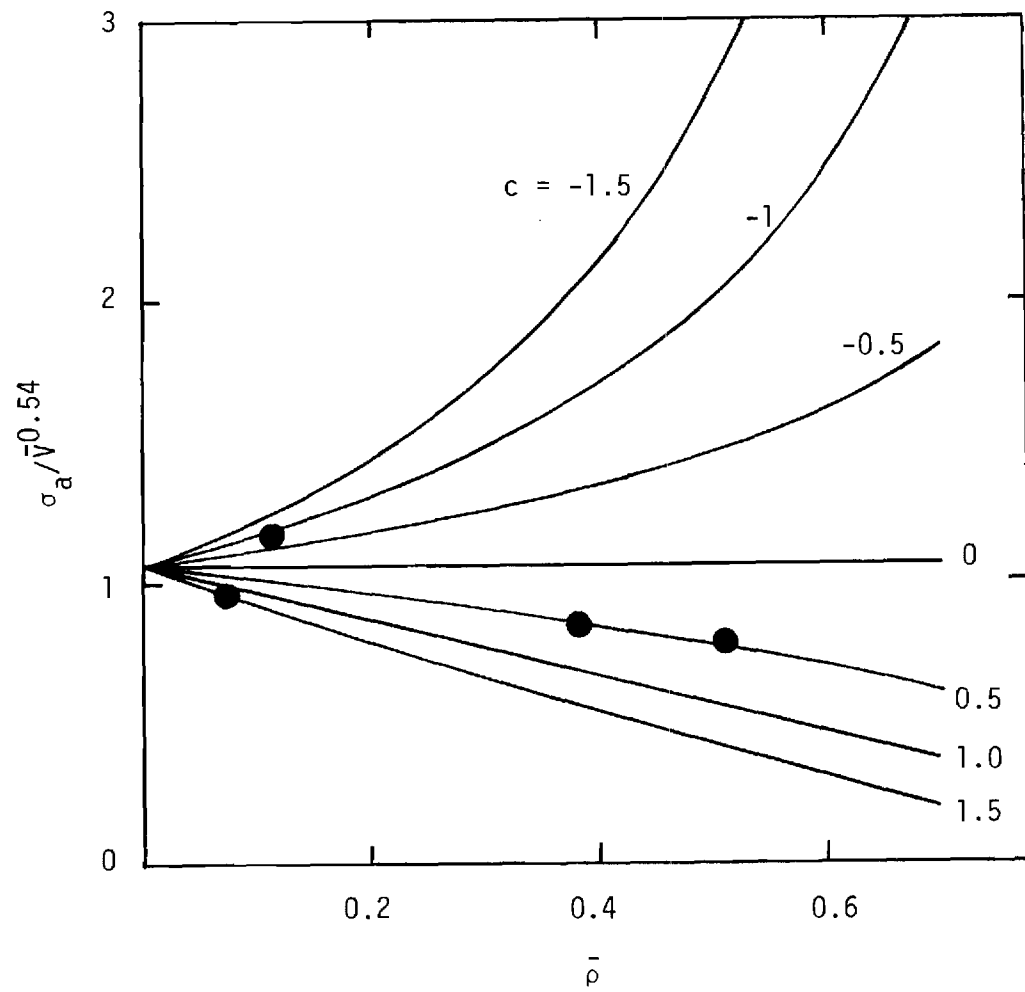


Figure C-1. Theoretical and Observed Ratio  $\sigma_a / \bar{V}^{0.54}$  Versus Correlation  $\rho$